

General Certificate of Education (A-level) January 2011

Mathematics
MFP1

## (Specification 6360)

Further Pure 1

Mark Scheme

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## Key to mark scheme abbreviations

| M | mark is for method |
| :---: | :---: |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| $\checkmark$ or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0 ) accuracy marks |
| $-x$ EE | deduct $x$ marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award full marks. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn no marks.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns full marks, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains no marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 1(a) <br> (b) <br> (c) | $\alpha+\beta=6, \alpha \beta=18$ <br> Sum of new roots $=6^{2}-2(18)=0$ <br> Product $=18^{2}=324$ <br> Equation $x^{2}+324=0$ <br> $\alpha^{2}$ and $\beta^{2}$ are $\pm 18 \mathrm{i}$ | $\begin{gathered} \text { B1B1 } \\ \text { M1A1F } \\ \text { B1F } \\ \text { A1F } \\ \text { B1 } \end{gathered}$ | 1 | ft wrong value(s) in (a) <br> ditto <br> ' $=0$ ' needed here; <br> ft wrong value(s) for sum/product |
|  | Total |  | 7 |  |
| 2(a) <br> (b)(i) <br> (ii) | $\begin{aligned} & \int_{q} 2 x^{-3} \mathrm{~d} x=-x^{-2}(+c) \\ & \int_{p}^{q} 2 x^{-3} \mathrm{~d} x=p^{-2}-q^{-2} \end{aligned}$ <br> As $p \rightarrow 0, p^{-2} \rightarrow \infty$, so no value <br> As $q \rightarrow \infty, q^{-2} \rightarrow 0$, so value is $1 / 4$ | M1A1 <br> A1F <br> B1 <br> M1A1F | 3 | M1 for correct index <br> OE; ft wrong coefficient of $x^{-2}$ <br> ft wrong coefficient of $x^{-2}$ or reversal of limits |
|  | Total |  | 6 |  |
| 3(a)(i) | $\begin{aligned} & {\left[\begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array}\right]} \\ & {\left[\begin{array}{cc} -1 & 0 \\ 0 & -1 \end{array}\right]} \end{aligned}$ | B1 <br> B1 | 1 1 |  |
| (b)(i) (ii) | $\begin{aligned} & \mathbf{A B}=\left[\begin{array}{cc} -20 & 14 \\ 14 & -10 \end{array}\right] \\ & \mathbf{A}+\mathbf{B}=\left[\begin{array}{cc} 0 & 5 \\ -5 & 0 \end{array}\right] \\ & (\mathbf{A}+\mathbf{B})^{2}=\left[\begin{array}{cc} -25 & 0 \\ 0 & -25 \end{array}\right] \end{aligned}$ | M1A1 <br> B1 <br> B1 | 2 | M1A0 if 3 entries correct |
|  | $\ldots=-25 \mathbf{I}$ | B1F | 3 | $\mathrm{ft} \mathrm{if} \mathrm{c's}(\mathbf{A}+\mathbf{B})^{2}$ is of the form $k \mathbf{I}$ |
| (c)(i) | Rot'n $90^{\circ}$ clockwise, enlargem't SF 5 | B2, 1 | 2 | OE |
| (ii) | Rotation $180^{\circ}$, enlargement SF 25 | $\mathrm{B} 2,1 \mathrm{~F}$ | 2 | Accept 'enlargement SF -25 '; <br> ft wrong value of $k$ |
| (iii) | Enlargement SF 625 | B2, 1F | 2 | B1 for pure enlargement; ft ditto |
|  | Total |  | 13 |  |
| 4 | $\begin{aligned} & \sin \left(-\frac{\pi}{6}\right)=-\frac{1}{2} \\ & \sin \left(-\frac{5 \pi}{6}\right)=-\frac{1}{2} \end{aligned}$ <br> Use of $2 n \pi$ <br> Going from $4 x-\frac{2 \pi}{3}$ to $x$ $\text { GS } x=\frac{\pi}{8}+\frac{1}{2} n \pi \text { or } x=-\frac{\pi}{24}+\frac{1}{2} n \pi$ | B1 <br> B1F <br> M1 <br> m1 <br> A1A1 | 6 | OE; dec/deg penalised at 6th mark OE ; ft wrong first value (or $n \pi$ ) at any stage including division of all terms by 4 OE |
|  | Total |  | 6 |  |

## MFP1(cont)

| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 5(a)(i) | $z_{1}{ }^{2}=\frac{1}{4}-\mathrm{i}+\mathrm{i}^{2}=-\frac{3}{4}-\mathrm{i}$ | M1A1 | 2 | M1 for use of $\mathrm{i}^{2}=-1$ |
| (ii) | LHS $=-\frac{3}{4}-\mathrm{i}+\frac{1}{2}+\mathrm{i}+\frac{1}{4}=0$ | M1A1 | 2 | AG; M1 for $z^{*}$ correct |
| (b) | LHS $=-\frac{3}{4}+i+\frac{1}{2}-i+\frac{1}{4}=0$ | M1A1 | 2 | AG; M1 for $z_{2}{ }^{2}$ correct |
| (c) | $z \text { real } \Rightarrow z^{*}=z$ <br> Discr't zero or correct factorisation | $\begin{gathered} \text { M1 } \\ \text { A1 } \end{gathered}$ | 2 | Clearly stated AG |
|  | Total |  | 8 |  |
| 6(a) | Sketch of ellipse | M1 |  | centred at origin |
|  | Correct relationship to circle | A1 |  |  |
|  | Coords $( \pm 2 \sqrt{2}, 0),(0, \pm \sqrt{2})$ | B2,1 | 4 | Accept $\sqrt{8}$ for $2 \sqrt{2}$; |
|  |  |  |  | B1 for any 2 of $x= \pm 2 \sqrt{2}, y= \pm \sqrt{2}$ allow B1 if all correct except for use of decimals (at least one DP) |
| (b)(i) | $\text { Replacing } x \text { by } \frac{x}{2}$ | M1 |  | or by $2 x$ |
|  | $E$ is $\left(\frac{x}{2}\right)^{2}+y^{2}=2$ | A1 | 2 |  |
| (ii) | Tangent is $\frac{x}{2}+y=2$ | M1A1 | 2 | M1 for complete valid method |
|  | Total |  | 8 |  |
| 7(a) | Denom never zero, so no vert asymp | E1 |  |  |
|  | Horizontal asymptote is $y=0$ | B1 | 2 |  |
| (b) | $x-4=k\left(x^{2}+9\right)$ | M1 |  |  |
|  | Hence result clearly shown | A1 | 2 | AG |
| (c) | Real roots if $b^{2}-4 a c \geq 0$ | E1 |  | PI (at any stage) |
|  | Discriminant $=1-4 k(9 k+4)$ | M1 |  |  |
|  | ... $=-\left(36 k^{2}+16 k-1\right)$ | m1 |  | m1 for expansion |
|  | $\ldots=-(18 k-1)(2 k+1)$ | m1 |  | m 1 for correct factorisation |
|  | Result (AG) clearly justified | A1 | 5 | eg by sketch or sign diagram |
| (d) | $k=-\frac{1}{2} \Rightarrow-\frac{1}{2} x^{2}-x-\frac{1}{2}=0$ | M1A1 |  | or equivalent using $k=\frac{1}{18}$ |
|  | $\ldots \Rightarrow(x+1)^{2}=0 \Rightarrow x=-1$ | A1 |  |  |
|  | $k=\frac{1}{18} \Rightarrow \frac{1}{18} x^{2}-x+\frac{9}{2}=0$ | A1 |  |  |
|  | $\ldots \Rightarrow(x-9)^{2}=0 \Rightarrow x=9$ | A1 |  |  |
|  | SPs are $\left(-1,-\frac{1}{2}\right),\left(9, \frac{1}{18}\right)$ | A1 | 6 | correctly paired |
|  | Total |  | 15 |  |

## MFP1(cont)



